Heligoland), R. D. Abell, B.Sc. (University of Leipzig), W. Caldwell, B.A. (University of Würzburg), W. B. McLean, B.Sc. (Owens College, Manchester), B. D. Steele, B.Sc. (University of Breslau), E. J. Butler, M.B. (University of Freiburg), J. W. Mellor, B.Sc. (Owens College, Manchester), L. N. G. Filon, M.A. (King's College, Cambridge). Four scholarships granted in 1898 have been exceptionally renewed for a third year. These scholars and their places of study are :—Dr. A. H. Reginald Buller, B.Sc. (University of Munich), H. T. Calvert, B.Sc. (University of Leipzig), R. L. Wills, B.A. (Cavendish Laboratory, Cambridge), E. H. Archibald, M.Sc. (Harvard University).

SCIENTIFIC SERIALS.

American Journal of Mathematics, vol. xxii. No. 3.—On continuous binary Λ linearoid groups, and the corresponding differential equations and Λ functions, by E. J. Wilczynski. In a previous paper (vol. xxi. 2) the writer has shown that, corresponding to every group of the form

$$\eta_i = \sum_{k=1}^{m} \phi_{ik}(x; a, \ldots a_r)\, y_k \quad (1),$$

where the r parameters a_i are essential, there exists a system of differential equations of order r, whose general solutions are given by (1), if $y_1, \ldots y_n$ form a fundamental system. The functions ϕ_{ik} were supposed to be uniform functions of x, and it was found that, if the parameters a_i were properly chosen, ϕ_{ik} were uniform functions of the parameters also. In the present paper he discusses these groups, the corresponding differential equations, and their solutions for the case when $n = 2$. Dr. Lovett, in his note on a property of lines in n-dimensional space, working on the lines of Cesàro's "Lezioni di Geometria Intrinseca," shows that a line of multiple curvature cuts its osculating space of highest dimensions, or lies wholly on one side of that space, according as the number of dimensions of the space necessary to the existence of the curve is odd or even.—Concerning the cyclic sub-groups of the simple group G of all linear fractional substitutions of determinant unity in two non-homogeneous variables with coefficients in an arbitrary Galois field, by Dr. L. E. Dickson (read before the Chicago section of the Mathematical Society, December 1899), leads to a generalisation to the GF[p^m] of results due to Prof. W. Burnside ("On a Class of Groups defined by Congruences," *Proc.* of London Math. Soc. vol. xxvi.). Variations from Burnside's method of treatment are introduced, partly to avoid the separate treatment of the cases $d=1$ and $d=3$, and to take in the exceptional cases $p=2$ and $p=3$, and to reduce the calculations; and further, on the other hand, to amplify some of the proofs. A few errors are also pointed out and amended.—On some invariant scrolls in collineations which leave a group of five points invariant, by V. Snyder. The writer gives numerous references to memoirs in which the quadric surfaces which are left invariant by cyclical collineations have been exhaustively treated. There is another simple series of scrolls, viz. those contained in a linear congruence, which have not been considered, except one form noticed by Ameseder. The writer confines his attention to such surfaces. There are six collineations which are of essentially different type, which project a set of five points into themselves without leaving every point invariant. In the notation of substitution-groups these may be thus represented :

$$T_2 \equiv (A_1A_2)(A_3)(A_4)(A_5),$$
$$T_3 \equiv (A_1A_2)(A_3A_4)(A_5),$$
$$T_4 \equiv (A_1A_2A_3)(A_4)(A_5),\ T_5 \equiv (A_1A_2A_3)(A_4A_5),$$
$$T_6 \equiv (A_1A_2A_3A_4)(A_5) \text{ and } T_7 = (A_1A_2A_3A_4A_5).$$

—On the reduction of hyperelliptic integrals ($p=3$) to elliptic integrals by transformations of the second and third degrees, by W. Gilespie. The point of the paper is an application of cubic involution to the problem of the reduction to elliptic integrals, of hyperelliptic integrals of genus $p=3$ and of the first kind, by a rational transformation of the third degree. It is a continuation of Prof. Bolza's researches on the cubic transformation ("Die Cubische Involution und Dreitheilung, &c.," and "Zur Reduction Hyperelliptischer Integrals, &c.," *Math. Ann.*, Bd. 50, pp. 68 and 314).—The closing paper, by Dr. E. H. Moore, was read before the American Mathematical Society at the Buffalo meeting of the summer of 1896, and is entitled "The Cross-ratio Group of n! Cremona Transformations of Order $n-3$ in Flat Space of $n-3$ Dimensions."

Bulletin of the American Mathematical Society, July.—Some remarks on tetrahedral geometry, by Dr. Timerding, is a paper read at the June meeting. Several properties of a tetrahedral complex are given, viz. the pole curves of such a complex of lines form again another such complex among the cubic space curves circumscribed about the fundamental tetrahedron, the complex curves of such a complex of lines form another tetrahedral complex, &c.—Prof. H. B. Newson's paper on singular transformations in real projective groups was read at the April meeting. It treats of transformations in real projective groups which can not be generated from the real infinitesimal transformations of certain continuous groups. The discussion, which is limited to one and two dimensions, can be readily extended to three and higher dimensions.—Miss Schottenfels, in a paper read at the June meeting, writes on groups of order 8!/2, and gives a simple proof of a correspondence established by Dr. Dickson (*Proc.* of London Math. Soc., vol xxx.).—Prof. F. S. Woods continues his notes on Lobachevsky's geometry.—Prof. Pierpont reviews H. Burkhardt's "Functionen-theoretische Vorlesungen" (vol. ii. "Elliptische Functionen").—A "correction," notes, new publications, list of papers read before the Society, with references to the places of their publication, and a full index, complete the sixth volume of the second series.

Annalen der Physik, No. 7.—Dispersion of electricity in air, by J. Elster and H. Geitel. Since the sun's rays contain ultra-violet light before they impinge upon the atmosphere, this light must ionise the upper strata, and the ions produced will be gradually distributed through the whole of the atmosphere by diffusion and convection. Hence the atmosphere will contain stray ions of both signs, but chiefly negative ones in the lower strata, owing to their superior mobility. The presence of these ions can be made evident by an electroscope.—Influence of slight impurities upon the spectrum of a gas, by P. Lewis. Very small quantities of hydrogen and nitrogen considerably affect the spectra of helium and argon, but the reverse is not the case.—Fluorescence and phosphorescence in the electric discharge through nitrogen, by P. Lewis. When nitrogen prepared from ammonium nitrate and sulphate, and purified over hot copper is pumped through an H-shaped vacuum tube, the whole wall of the tube shows a brilliant fluorescence lasting a few seconds, which extends for a length of about a yard into the supply and exhaust tubes. The light can be made permanent by keeping the pump at work and thus supplying a continuous stream of fresh nitrogen. Spectroscopic examination shows that the fluorescence is dependent upon the presence of a number of bands in the extreme ultra-violet, due to a combination of nitrogen with a trace of oxygen.—Production of very high notes by Galton's whistle, by M. T. Edelmann. The author gives tables for the pitches of pipes of various dimensions, and instructions how to test the pipes by Kundt's dust figures. He has succeeded in constructing a pipe of only 2 mm. diameter, which gives the enormously high pitch of 170,000 complete vibrations per second, or over two octaves beyond the extreme limit of audibility.—The magnetic force of the atoms, by R. Lang. Magnetism is accounted for by the revolutions of negative about positive electrons.—The air thermometer at high temperatures, by L. Holborn and A. Day. The authors further investigate the properties of the air thermometer consisting of a platinum-rhodium vessel filled with nitrogen, and compare its indications with that of a platinium-iridium thermo-couple, paying particular attention to their regular expansion of the vessel. The corrected value for the melting point of gold is 1064·0° C.—Difference of temperature between the surface and the interior of a radiating body, by F. Kurlbaum. A method is given of determining this difference of temperature by means of two bolometers exposed symmetrically to different surfaces of the same black partition.

SOCIETIES AND ACADEMIES.

LONDON.

Royal Society, June 14.—"Data for the Problem of Evolution in Man. V. On the Correlation between Duration of Life and the Number of Offspring." By Miss M. Beeton, G. U. Yule, and Karl Pearson, F.R.S., University College, London.

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According to the Darwinian theory of evolution the members of a community less fitted to their environment are removed by death. But this process of natural selection could not permanently modify a race if the members thus removed were able before death to propagate their species in average numbers. It then becomes an important question to ascertain how far duration of life is related to fertility. In the case of many insects death can interfere only with their single chance of offspring; they live or not for their one breeding season only. A similar statement holds good with regard to annual and biennial plants. In such cases there might still be a correlation between duration of life and fertility, but it would be of the indirect character, which we actually find in a case of men and women living beyond sixty years of age—a long life means better physique, and better physique increased fertility. On the other hand, there is a direct correlation of fertility and duration of life in the case of those animals which generally survive a number of breeding seasons, and it is this correlation which we had at first in view when investigating the influence of duration of life on fertility in man. The discovery of the indirect factor in the correlation referred to above was therefore a point of much interest. For it seems to show that the physique fittest to survive is really the physique which is in itself (and independently of the duration of life) most fecund.

The data dealt with in this paper consists of four series, the first three collected and reduced by Miss M. Beeton, and the fourth series by Mr. G. U. Yule.

Mothers. Length of Life and Size of Family.

Series I.—Taken from the "Whitney Family of Connecticut," a well-known history of an American Quaker family.

Series II.—Taken from purely English Quaker records. The data for this series were drawn from a great variety of histories and records of the Society of Friends.

Fathers. Length of Life and Size of Family.

Series III.—The great bulk of the data was extracted from the American Whitney Family.

Series IV.—Extracted from Burke's "Landed Gentry."

The following are some of the chief results obtained from the reduction of these series:—

Table of General Results.

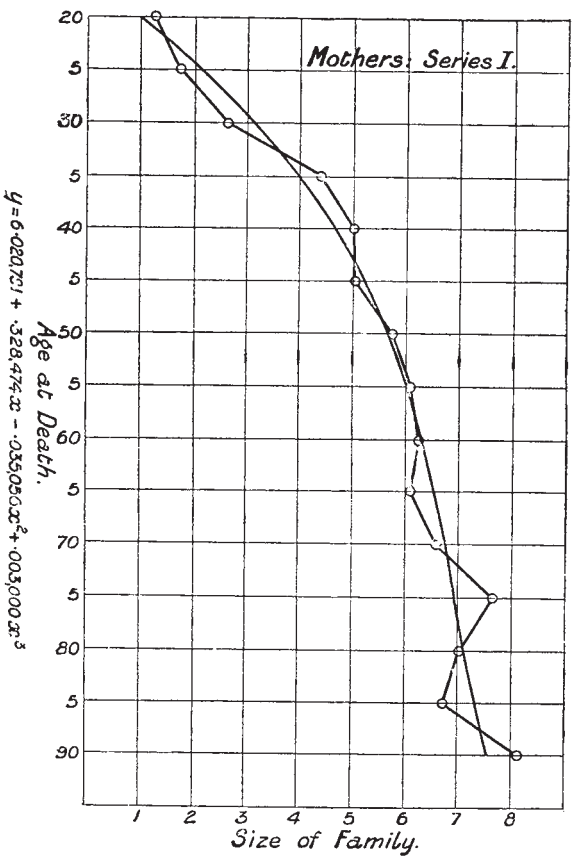
Series.	Parent.	Mean age at death.	Mean size of family.	Correlation fertility and duration of life.
I.	Mother	53·292	5·269	0·4943
II.	Mother	61·183	5·811	0·2340
III.	Father	58·086	5·469	0·4764
IV.	Father	63·577	5·336	0·2010

It is shown that the peculiar physique in both men and women which leads to longevity is also associated with greater fecundity. Of two women who both live beyond fifty years, the longer lived is likely to have had before fifty the larger family. The association is, however, much greater for American than English parents, although the American parents dealt with are, in the great majority of cases, of Anglo-Saxon race. Climate, mode of life, in general selection and environment, seem to be differentiating in this respect the English and the Anglo-American. The English Friends, we should suppose, would be as a class very comparable with the American Friends; yet their average life is longer, their fertility greater, and there is less association between longevity and fecundity. In both cases our algebraical formulæ show that American men and women are more alike, and English men and women are more alike than the women to the women or the men to the men of the two races. This is the more remarkable, as the English Friends as a class are by no means identical with the Landed Gentry.

In order to represent the *continuous* change in the regression, which cannot be done by two straight lines, which only enable us to distinguish the fecund and non-fecund periods of life, the statistics were fitted with cubical parabolas. The regression line at any age in life may then be looked upon as the tangent to the cubical parabola at that age. An inspection of the diagram given below for American mothers shows what an excellent expression such parabolas are for these statistics

For American mothers and fathers we see dy/dx consistently positive throughout life, and we have a most excellent graphical demonstration of the physical characters which tend to longevity being also associated with fecundity.

Weismann has suggested that it may be an advantage to a species that its duration of life should be shortened. This is not, *a priori*, confirmed by the case of a man in the American series: the longer the parents live, the greater the number of their offspring. But if we can lay any stress on a bend-in for the English mothers, and on a similar, but less marked, tendency for the English fathers, we might argue that reproductive selection was possibly in England working against extreme longevity, although favouring parents living till sixty-five or seventy. Indeed, those who rush rapidly to brilliant, but not



over-stable, conclusions might emphasise Weismann's views by showing how in an old community, with much greater pressure on the material resources, there is a tendency to reduce the fertility of the long-lived parents; while in a new community, with plenty of food and occupation for all, the longest-lived parents are the most fertile! However, all that we can safely say is that there is a marked difference between English and American parents, and that this distinguishing characteristic is almost equally visible if we take opposite sexes of such diverse classes as English Friends and English country gentlemen. We would leave to further investigations its true interpretation.

Admitting a substantial correlation between length of life and fertility, it is of great interest to investigate what effect, other things being equal, reproductive selection would have in modifying the duration of life.

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The following table gives the mean length of life of parents taken singly and of parents weighted with their offspring :—

Mean Duration of Life of Parents in Years.

Series.	Unweighted parents.	Weighted parents.	Progression.
I.	53·292	59·920	6·628
II.	61·183	63·839	2·656
III.	58·086	63·082	4 996
IV.	63·577	65·510	1·930

Now these are substantial differences even in the case of the English parents, but they are very large differences in the case of the American parents. If we suppose no assortative mating on the basis of characters tending towards longevity, then it is easy to measure by a rough approximation the effect of reproductive selection in modifying the duration of life.

The increased duration of life would be about two years per generation from the American data, and about 9 to 9·5 months per generation from the English data.

The result for the American series shows us how an especially low expectation of life, due possibly in this case to some family character, will be rapidly raised by reproductive selection, if there be no opposing factor of evolution. The English results on the other hand show us a small, but sensible, tendency in reproductive selection to prolong the duration of life. Allowing three generations to a century, we might expect the duration of life to be raised about two years in a century by this factor of evolution.

A somewhat widespread view of evolution stops at the survival of the fitter without discussing the mode whereby the less fit leave no, or fewer, offspring than the fit. Of course, if the unfit are exterminated before adult life, there is no chance of their reproducing themselves. It has been shown in "Data for the Problem of Evolution in Man (II.)" that a selective death-rate exists for adults, so that the whole work of selection does not take place before the reproductive stage is reached. But Miss Beeton's data for the correlation of duration of life in the case of brethren dying as minors seem to show that the selective death-rate for children is rather less, not greater, than its value for adults.[1] Hence, for the reduction or extermination of stock unsuited to its environment, we should have to look largely to selection in the adult state. In the present paper we have made what we believe to be the first quantitative determination of how a selective mortality reduces the numbers of the offspring of the less fit relatively to the fitter. In the case of life under wild conditions, the correlation between fertility and power of surviving would probably be far greater. But for such life it is almost impossible to get statistics of this nature; we are thrown back upon measuring the effect in man, and thus obtaining what may well be considered as a minimum value of the influence under discussion.

In the course of our investigations we have found that the relationship between fertility and duration of life does not cease with the fecund period. We thus reach the important result that characters which build up a constitution fittest to survive are also characters which encourage its fertility. This result is of great value from the standpoint of the differentiation of type, where it is absolutely necessary that the fittest to survive should also be the most fertile. On the other hand, we note that duration of life is a character capable of modification by reproductive selection, and we suggest that a considerable part of the increased expectation of life observed in recent years may be due to this cause. In the case of the American statistics, we see at once how reproductive selection can replace a remarkably short-lived stock by a longer-lived stock, for the bulk of the offspring come from the longer-lived members.

PARIS.

Academy of Sciences, July 30.—M. Maurice Lévy in the chair.—On the observatory at Mount Etna, by M. J. Janssen. Remarks on local difficulties due to climatic disturbances and to the peculiar situation of the observatory.—New processes of vaccination against symptomatic carbuncle of the ox by means of preventive serum in association with vaccines, by M. S. Arloing. A continuation of former experiments on the subject.—On the age of the sea-shore sands of Dunkirk, by M. J. Gosselet. The formation of these deposits is considered to have commenced since the fourth or fifth centuries.—M. Duhem was elected a corresponding member for the section of mechanics.—Observations of Borrelly's comet (July 23, 1900) at the Paris Observatory, by M. G. Bigourdan.—Provisional elements and ephemerides of the Borrelly-Brooks' comet (July 23, 1900), calculated by M. G. Fayet.—On the spectral images of the chromosphere and protuberances obtained with the prismatic chamber, by M. Georges Meslin. A description of the results obtained with the apparatus previously described.—On two surfaces related to every Weingarten's surface, by M. A. Demoulin.—On artificial radio-active barium, by M. A. Debierne. Many substances become radio-active when brought into intimate contact, by solution or simultaneous precipitation, with radio-active compounds. Artificial radio-active barium chloride, intermediate in character between barium and radium, has thus been obtained.—On the thermo-electricity of steels, by M. G. Belloc. A comparative study of the thermo-electric properties of soft iron, soft steel, and hard steel.—On a means of weakening the influence of industrial electric currents on the terrestrial field in magnetic observatories, by M. Th. Moureaux. An account of the methods whereby the disturbances caused by electric tramways in the neighbourhood of observatories may be removed or corrected for.—On the electrolysis of concentrated solutions of hypochlorites, by M. André Brochet. The electrolysis of hypochlorite resembles, in its later stages, that of alkaline chloride solutions, and tends towards the same limits. There is, therefore, little hope of obtaining concentrated solutions of hypochlorites by the direct electrolysis of chlorides. —On gadolinium, by M. Eug. Demarçay. A study of the spectrum of gadolinium —On diphenylcarbazide as a sensitive reagent for some metallic compounds, by M. P. Cazeneuve. The conversion of diphenylcarbazide into diphenylcarbazone by the action of salts of copper and mercury and the persalts of iron, as recently described, furnishes a delicate test for these metals. The latter unite with the carbazone to form coloured compounds.—Preliminary study of the chemism of the encephalon, by M. N. Alberto Barbieri. Experiments on the chemical changes occurring in the brain of animals when left for twelve to eighteen hours at a temperature of 45°. —On the dissolution of the nitrogenous constituents of malt, by MM. P. Petit and G. Labourasse. Experiments relating to the existence of a proteolytic enzyme in malt.—Action of the liquid from the external prostate of the hedgehog on the liquid of the seminal vesicles; nature of this action, by MM. L. Camus and E. Gley.—On some properties and reactions of the liquid from the internal prostate of the hedgehog, by MM. L. Camus and E. Gley. This and the previous paper form a continuation of the authors' researches on the coagulation of the secretion of the seminal vesicles by that of the external prostate, or Cooper's gland. and the coagulation of the latter secretion by that of the internal prostate.—On some Alpheidæ of the American coasts, by M. H. Coutière. An account of some specimens in the collection at the United States National Museum, Washington.

August 6.—M. Maurice Lévy in the chair.—The menstrual function and rut in animals. Function of arsenic in the economy, by M. Armand Gautier. The author has found that the quantities of arsenic and iodine, which in normal blood are hardly estimable, are largely increased during menstruation, the total amount of arsenic eliminated during one period of menstruation representing the whole amount usually present in the thyroid gland. The arsenic and iodine which accumulate in the thyroid gland are eliminated in the male by the hair and nails, and by epithelial desquamation. In the female, this excess is either eliminated by the genital organs or utilised by the growing fœtus.—Observations of the star Capella, considered as a double star, made at the Greenwich Observatory, by Mr. W. H. M. Christie. The independent discovery by Campbell and Newall, by spectroscopic observations, that Capella is a double star, has been confirmed by direct observation with the large Greenwich equatorial. The star appears distinctly elongated in one direction, the distance of the two components being estimated at 0·1 second. Observations of the direction of this elongation, taken between April 4 and July 20, confirm

[1] The point is still under investigation.

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the period of revolution deduced spectroscopically by Newall.—The comet (1900 *b*) discovered July 23 at the Observatory of Marseilles, by M. Borrelly. The comet is visible to the naked eye as a star of 6th to 7th magnitude.—Observations of the comet 1900 *b* (Borrelly-Brooks) made at the Observatory of Besançon, by MM. A. Sallet and P. Chofardet.—Observations of the Borrelly-Brooks comet, made at the Toulouse Observatory with the 25 cm. equatorial, by M. F. Rossard.—On circuits formed uniquely by electrolytes, by MM. Camichel and Swyngedauw. From the experiments described with circuits consisting wholly of liquid electrolytes, the authors conclude that an electrolyte may be traversed by a current without decomposition.—On the coupling up of alternators from the point of view of the harmonics, and of the effect of synchronised motors, by M. A. Perot.—On the boiling points of zinc and cadmium, by M. Daniel Berthelot. The metals were boiled in an electric furnace specially constructed to avoid the errors due to superheating and radiation from the walls, the temperature being measured by the interference refractometer method previously described by the author. Zinc boiled at 920°, and cadmium at 778°.—On the atomic weight of radiferous barium, by Mme. Curie. First attempts at determining the atomic weight of the metal in radiferous barium chloride gave 146 as against 138 for pure barium chloride. As the result of prolonged fractionations, a product has now been obtained in which the atomic weight is as high as 174. This, however, is certainly too low, as the chloride analysed still contains an unknown amount of barium.—On the electrolytic estimation of cadmium, by M. Dmitry Balachowsky. The metal is deposited upon a dish previously covered with copper. The solution is slightly acidified with nitric acid, and the deposition carried out at 60° under conditions of electromotive force and current density specified.—On some new spectra of rare earths, by M. Eug. Demarçay.—On the blue oxide of molybdenum, by M. Marcel Guichard. The hydrated blue oxide of molybdenum has been isolated in a pure state and analysed, and proved to have the composition $MoO_2.4MoO_3,6H_2O$.—On the normal proportions of iodine in the organism, and its elimination, by M. P. Bourcet. The author, in conjunction with M. Gley, having previously shown the presence of a trace of iodine in normal blood, has now determined the amount of this element in various parts of the body. The quantities found vary from 0·00 mgr. in fat, pancreas and bladder, to 0·18 mgr. per 100 grams of liver and 1·8 mgr. per 100 grams of hair. The quantities found are small compared to the amount present in the thyroid gland. About 0·33 mgr. of iodine is taken into the human system daily in food; the thyroid gland contains only about 4 mgr.; hence it becomes necessary to discover the means of elimination. This is shown to be chiefly effected in man by the skin and epidermal products, sweat, skin, hair and nails; in women, by the menstrual blood, which contains 0·8 to 0·9 mgr. of iodine per kilogram, as against 0·02 mgr. per kilogram in normal blood.—On the nitrogenous substances in malt, by MM. P. Petit and G. Labourasse.—On the origin of the secondary calcareous breccia of Ariege, and results drawn from the point of view of the age of the lherzolite, by M. A. Lacroix.—On some temperatures observed in the park of St. Maur, by M. E. Renou.

New South Wales.

Royal Society, June 6.—The President, Prof. Liversidge, F.R.S., in the chair.—On the relation, in determining the volumes of solids, whose parallel transverse sections are n^{ic} functions of their position on the axis, between the position and coefficients of the section and the (positive) indices of the function, by G. H. Knibbs.—On the amyl ester of eudesmic acid occurring in eucalyptus oils, by Henry G. Smith. In a paper read before this society, July 1898, on the stringy-bark trees of New South Wales, R. T. Baker and the author show that an ester was present in the oil of *Eucalyptus macrorhyncha*. Since then esters have been found to be present in several eucalyptus oils. The author shows that esters are present in fair amount in the oils of *E. botryoides*, *E. Saligna*, and *E. rostrata*, and that an aromatic alcohol, either linalool or geraniol, is present in the oil of *E. patentinervis*, over 16 per cent. of free alcohol being proved. The saponified oil of *E. patentinervis* has a fine odour. Citral also occurs in this oil, proved by its characteristic reactions.—Note on a new meteorite from New South Wales, by R. T. Baker. The meteorite described in this paper was found early in January of this year, two miles from Bugaldi, a a postal town fifteen miles north-west of Coonabarabran. It is pear-shaped and is nearly five inches long and three inches wide at the broadest part. It belongs to that class of meteorites known as siderites, and is probably composed of iron and nickel. It has a well-defined, closely adhering "skin" of black magnetic material, while the metal immediately beneath this coating is silvery-white in appearance. On the smooth portion at the extremity of the larger end can be seen very distinctly Widmanstatten's figures. The specimen has an exceedingly new appearance, as if it had only just arrived upon the earth, and shows no signs of oxidation.

Göttingen.

Royal Society of Sciences.—The *Nachrichten* (physico-mathematical section), Part i. for 1900, contains the following memoirs communicated to the Society.

February 3.—L. Krüger: Compensation of errors by means of equations of condition in geodetic determinations of points.

March 3.—E. Marx: Fall of potential and dissociation in flame-gases.—W. Nernst: On the question of the hydratation of dissolved substances, Part 1.—H. Lotmar: The same, Part 2.—C. C. Garrard and E. Oppermann: The same, Part 3.—H. Minkowski: Theory of the units in algebraic *Zahlkörper*.

March 16.—W. F. Osgood: On a theorem of Schönflies relating to the theory of the functions of two real variables.—F. Bernstein: On the same theorem.—H. E. Timerding: On linear systems of conics.

Among the official reports of the Society are one (by Prof. F. Klein) on the publication of Gauss's works; and one on the progress of the Encyclopædia of Mathematics.

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